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The current-voltage relationship revisited: exact and approximate formulas with almost general validity for hot magnetospheric electrons for bi-Maxwellian and kappa distributions

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Abstract. We derive the current-voltage relationship in the auroral region taking into account magnetospheric electrons for the bi-Maxwellian and kappa source plasma distribution functions. The current-voltage formulas have in principle been well known for a long time, but the kappa energy flux formulas have not appeared in the literature before. We give a unified treatment of the bi-Maxwellian and kappa distributions, correcting some errors in previous work. We give both exact results and two kinds of approximate formulas for the current density and the energy flux. The first approximation is almost generally valid and is practical to compute. The first approximation formulas are therefore suitable for use in simulations. In the second approximation we assume in addition that the thermal energy is small compared to the potential drop. This yields even simpler linear formulas which are suitable for many types of event studies and which have a more transparent physical interpretation than the first approximation formulas. We also show how it is possible to derive the first approximation formulas even for those distributions for which the exact results can not be computed analytically. The kappa field-aligned conductance value turns out always to be smaller than the corresponding Maxwellian conductance. We also verify that the obtained kappa current density and energy flux formulas go to Maxwellian results when $\kappa \to \infty$.

Key words. Current-voltage relationship · Bi-Maxwellian distributions · Kappa distribution

1 Introduction

The purpose of this paper is to revisit the question of the current-voltage relationship and the energy flux formulas in the auroral region from the single-particle viewpoint. Possible applications of the formulas include global magnetohydrodynamic (MHD) simulations (Janhunen, 1996) which have ionospheric coupling included, as well as all studies where rocket, low-orbiting satellite or ground-based radar data are used to infer magnetospheric parameters (density, temperature) or ionosphere-magnetosphere coupling parameters (the potential drop, the field-aligned current and the field-aligned conductance) (e.g., Lyons *et al*., 1979; Lu *et al*., 1991; Olsson *et al*., 1996, 1997; Olsson and Janhunen, 1997).

Our initial development mainly follows Fridman and Lemaire (1980) (henceforth referred to as FL80), who used adiabatic single-particle theory to calculate the current-voltage relationship as well as the relationship between the voltage and the energy flux. We limit ourselves to discussing hot magnetospheric electrons only, which is a good approximation unless one is interested in potential drops much below 100 eV. In fact, for very small potential drops the ionospheric electron and ion populations should also be taken into account (Lemaire and Scherer, 1973, 1983; Pierrard, 1996). We give our results in terms of the magnetospheric source plasma density, not the density found at lower altitudes as was done by Pierrard (1996). We also neglect gravitation, which is a good approximation for electrons. Otherwise our assumptions are the same as those listed in FL80.

Our main interest is the case when the field-aligned current (FAC) is upward. For downward FAC regions our results will not hold, strictly speaking, but then the current-voltage relationship is simply $V = 0$ if we ignore anomalous resistivity. In ionosphere-magnetosphere coupling simulations we typically need to compute the "inverse" current-voltage relationship, i.e. the voltage as a function of the current. Thus no numerical problems arise in the downward current region even though we are dealing with infinite field-aligned conductance. In the upward FAC regions, however, using formulas such as the full Knight formula (Knight, 1973; Lemaire and Scherer, 1973) would be difficult because it would

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require numerical root finding at every point at every time-step; thus approximations are needed.

We show that the exact nonlinear current density formula, which was first derived for the case of Maxwellian distribution by Knight (1973), can be approximated in almost all practical situations by linearization with respect to the small quantity B_m/B_i, where B_m and B_i are the magnetospheric and ionospheric magnetic fields, respectively. We refer to this as the first approximation. Only for extremely large potential drops (more than 100 kV) does the first approximation become invalid, but it is very probable that these situations never arise in practice. In the case of a bi-Maxwellian distribution, this linearization has been done by previous authors (Lundin and Sandahl, 1978; FL80). However, these authors also made the further assumption that the potential drop is much larger than the thermal energy, which yields the well-known linear current-voltage relationship. In this paper we call this the second approximation.

The first and second approximation schemes can be defined not only for the current density but for the energy flux as well. In the case of a Maxwellian distribution, the energy flux formulas in the first and second approximation have appeared in the literature (Menietti and Burch, 1981) but the kappa distribution formulas have not. The kappa distribution (Vasyliunas, 1968) can model a high-energy tail in the precipitating electron flux.

As far as simulation work is concerned, the second approximation is not appropriate because it is only valid within auroral activity. However, in observational studies of substorm-related events the second approximation is usually valid, and has been used extensively.

We also present a simplified method by which the first and second approximation formulas can be derived for distributions more complicated than the bi-Maxwellian without having to get the exact results for the current density and the energy flux first. We apply this method to the kappa distribution and give the first and second approximation formulas for the kappa energy flux, which is a new result. An exact formula for the kappa energy flux is probably not possible to give in terms of known special functions, or at least the result would be extremely complicated. Finally, we correct some errors in previous studies.

2 Theory

In this study we consider the bi-Maxwellian distribution, given by

$$f_{BM}(W_\parallel, W_\perp) = N_e\left(\frac{m}{2\pi}\right)^{3/2}\frac{1}{T_\perp\sqrt{T_\parallel}}\exp\left(-\frac{W_\parallel}{T_\parallel}-\frac{W_\perp}{T_\perp}\right) \quad . \quad (1)$$

We also consider the kappa distribution given by

$$f_\kappa(W_\parallel, W_\perp) = \frac{1}{2\pi}N_e A_\kappa\left(\frac{m}{2\kappa T}\right)^{3/2}\left(1+\frac{W_\parallel + W_\perp}{\kappa T}\right)^{-(\kappa+1)} . \quad (2)$$

(Vasyliunas, 1968) where the normalization constant A_κ is given by $A_\kappa = \Gamma(\kappa+1)/[\Gamma(\kappa-1/2)\Gamma(3/2)]$. In these formulas, Γ is the Euler gamma function, N_e is the source plasma density (in the magnetosphere), $W_\parallel$ and $W_\perp$ are the parallel and perpendicular particle kinetic energies, m is the particle mass (the electron mass in this paper), and $T_\parallel$ and $T_\perp$ are the source plasma parallel and perpendicular temperature (in energy units). In the case of the kappa distribution we have only a single temperature parameter T_j κ is the parameter characterizing the kappa distribution (the power-law spectral index). Actually, if the true temperature $T_\kappa^{\rm true}$ of the kappa distribution plasma is defined in terms of the total energy density, it becomes $T_\kappa^{\rm true} = \kappa T/(\kappa - 3/2)$ (Collier, 1995, Olsson and Janhunen, 1997), where T is the parameter appearing in Eq. (2). We choose to write our formulas in terms of T rather than $T_\kappa^{\rm true}$.

Distributions given in Eqs. (1) and (2) are given in energy variables and are normalized to the particle number density N_e as

$$2\pi\int_0^\infty\int_0^\infty f(W_\parallel, W_\perp)\sqrt{\frac{2}{m^3 W_\parallel}}\,dW_\parallel\,dW_\perp = N_e \quad , \quad (3)$$

where f stands for either f_{BM} or f_κ. The current density at the ionospheric plane is computed from

$$j = 2\pi e\left(\frac{1}{2}\right)\left(\frac{B_i}{B_m}\right)\int_0^\infty\int_0^{W_\perp^{\max}} f(W_\parallel, W_\perp)\sqrt{\frac{2}{m^3 W_\parallel}}v_\parallel\,dW_\perp\,dW_\parallel \quad (4)$$

(FL80), where $v_\parallel = \sqrt{2W_\parallel/m}$ and $W_\perp^{\max}$ is given by

$$W_\perp^{\max} = \frac{T_\perp}{T_\parallel}x(W_\parallel + eV) \quad (5)$$

where

$$x = \frac{T_\parallel}{T_\perp}\frac{1}{B_i/B_m - 1} \quad (6)$$

and V is the ionosphere-magnetosphere potential difference (according to our convention, V is positive when the ionosphere is at a higher potential than the magnetosphere, i.e. when electrons precipitate). B_i and B_m are the magnetic field strengths at the ionosphere and the magnetospheric source plasma region, respectively.

Equation (4) is the same as the normalization integral [Eq. (3)], except that the factor $ev_\parallel$ has been added, the domain of integration has been reduced (the parameter $W_\perp^{\max}$) as explained in FL80, the factor (B_i/B_m) has been added to get the current density at the ionospheric plane, and the factor $(1/2)$ has been added to take into account the FAC into one hemisphere only.

The energy flux at the ionospheric plane is

$$\varepsilon = 2\pi\left(\frac{1}{2}\right)\left(\frac{B_i}{B_m}\right)\int_0^\infty\int_0^{W_\perp^{\max}} f(W_\parallel, W_\perp)\sqrt{\frac{2}{m^3 W_\parallel}} \times v_\parallel(W_\parallel + W_\perp + eV)dW_\perp dW_\parallel \quad . \quad (7)$$

This is similar to the current density formula of Eq. (4), except that now we drop the e and include $W_\parallel + W_\perp + eV$. Notice that the accelerating potential term eV must be added here to get the energy flux at the ionospheric level.

2.1 Bi-Maxwellian distribution

For the bi-Maxwellian distribution [Eq. (1)] the general current density formula given by Eq. (4) yields

$$j^{BM} = e\left(\frac{B_i}{B_m}\right)N_e\sqrt{\frac{T_\parallel}{2\pi m_e}}\left[1 - \frac{\exp(-xeV/T_\parallel)}{1+x}\right] \; . \tag{8}$$

This is the same formula as Eq. (5) in FL80, except that we are calculating the current density, not the particle flux.

The energy flux formula, Eq. (7), for the bi-Maxwellian distribution gives the result

$$\varepsilon^{BM} = \left(\frac{B_i}{B_m}\right)N_e\sqrt{\frac{T_\parallel}{2\pi m}}\Bigg\{T_\parallel + T_\perp + eV - e^{-xeV/T_\parallel}\left[T_\perp + eV\left(1 + x\frac{T_\perp}{T_\parallel}\right) + \frac{T_\parallel + xT_\perp}{(1+x)^2}\right]\Bigg\} \tag{9}$$

which is the same as Eq. (6) of FL80.

These expressions can be much simplified by invoking the approximation $x \ll 1$, where x is defined by Eq. (6). Usually this is a very good approximation, since the ionospheric magnetic field B_i is much larger than the magnetospheric magnetic field B_m. A straightforward series expansion of Eq. (8) in x yields

$$j^{BM}_{\text{Appr.}} = \left(\frac{T_\parallel}{T_\perp}\right)\frac{eN_e}{\sqrt{2\pi m T_\parallel}}\left(T_\parallel + eV\right) + O\left(x^2\right) \; . \tag{10}$$

This is the bi-Maxwellian current density formula in the first approximation. This is the formula currently in use in our ionosphere-magnetosphere coupling simulation (Janhunen, 1996). For large accelerating potentials ($eV \gg T_\parallel$), Eq. (10) can be further approximated as

$$j = \left(\frac{T_\parallel}{T_\perp}\right)\frac{e^2N_e}{\sqrt{2\pi m T_\parallel}}V \; . \tag{11}$$

which is the well-known linear current-voltage relationship (Lundin and Sandahl, 1978; FL80), which we call the second approximation.

The first approximation for the energy flux yields

$$\varepsilon^{BM}_{\text{Approx.}} = \frac{T_\parallel}{T_\perp}\frac{N_e}{\sqrt{2\pi m T_\parallel}}\left[2T_\parallel^2 + 2eVT_\parallel + (eV)^2\right] + O\left(x^2\right) \; . \tag{12}$$

Again, for accelerating potentials much larger than the thermal energies we obtain

$$\varepsilon = \left(\frac{T_\parallel}{T_\perp}\right)\frac{e^2N_e}{\sqrt{2\pi m T_\parallel}}V^2 \; , \tag{13}$$

which is the second approximation bi-Maxwellian energy flux formula.

Usually in the preceding formulas one does not know the parallel and perpendicular temperatures separately, so they are assumed equal, but we have given the more general expressions for reference purposes. Our expressions are in agreement with those given by FL80 [their Eqs. (9) and (10)]. Notice that in deriving Eqs. (10) and (12) our only assumptions were $x \ll 1$ and $x \ll T_\parallel/(eV)$, which are usually valid in all practical upward current cases, except possibly those having an extremely large potential drop. In particular, these formulas are valid for small potential drops also, as far as hot magnetospheric electrons as concerned. For very small potential drops (less than 100 eV) the ionospheric plasma source should also be taken into account, as was taken into account in the pioneering work of Pierrard (1996).

In Fig. 1 we compare the first and second approximated current densities, Eqs. (10) and (11), with the exact formula, Eq. (8). The parameters employed are listed in Table 1. The first approximation (dashed line) is indistinguishable from the exact result (solid line) for potential drops less than about 100 kV. The second approximation (dotted line) is notably different for small potential drops. Both first and second approximation are the same for large potential drops.

A similar comparison for the energy flux is shown in Fig. 2. In this case the first approximation and the exact result curves completely overlap.

2.2 Kappa distribution

For the kappa distribution, Eq. (2), the general current density formula, Eq. (4), yields

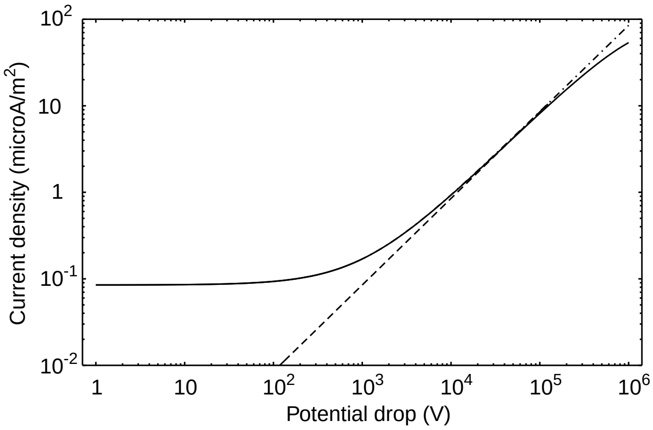

Fig. 1. The exact nonlinear current density (*solid*), the first approximation (*dash*) and the second approximation (*dot*) line for an isotropic Maxwellian distribution for parameters shown in Table 1. The exact and first approximation curves differ only for potential drops larger than about 100 kV. The first and second approximations, on the other hand, overlap for potential drops larger than about 30 kV, resulting in a *dash-dot line*.

Table 1. Parameters used in the plots

parameter	value
B_i	50000 nT
B_m	50 nT
N_e	0.1 cm^{-3}
$T = T_\perp = T_\parallel$	1 keV

$$j^\kappa = e\left(\frac{B_i}{B_m}\right) N_e \sqrt{\frac{T}{2\pi m}}\left[1 - \frac{1}{\left(1+\frac{eVx}{\kappa T}\right)^{\kappa-1}(1+x)}\right] \times \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\kappa^{1/2}(\kappa-1)} \quad . \tag{14}$$

In Fig. 3 we plot Eq. (14) for different kappa values. For large kappa values these curves tend to the Maxwellian result. For potential drops smaller than a few kV, smaller kappa values give larger current densities, as was also found by Pierrard (1996).

It can be shown that $\lim_{\kappa\to\infty} j^\kappa = j^{\mathrm{BM}}$, as Pierrard (1996) did for her formulas.

Expanding Eq. (14) to first order in x gives

$$j^\kappa_{\mathrm{Approx.}} = \frac{eN_e}{\sqrt{2\pi m T}}\left(T + \frac{\kappa-1}{\kappa} eV\right) \times \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\kappa^{1/2}(\kappa-1)} \quad , \tag{15}$$

which is the first approximation current density for kappa distribution. Further, if we assume that the potential drop is larger than the thermal energy ($eV \gg T$) we obtain the second approximation

$$j = \frac{e^2 N_e}{\sqrt{2\pi m T}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\kappa^{3/2}} V \quad . \tag{16}$$

In Fig. 4 we compare the first and second approximations, Eqs. (15) and (16), with the exact kappa current density formula, Eq. (14). The result is qualitatively similar to Fig. 1, where we made the same kind of comparison for the Maxwellian distribution.

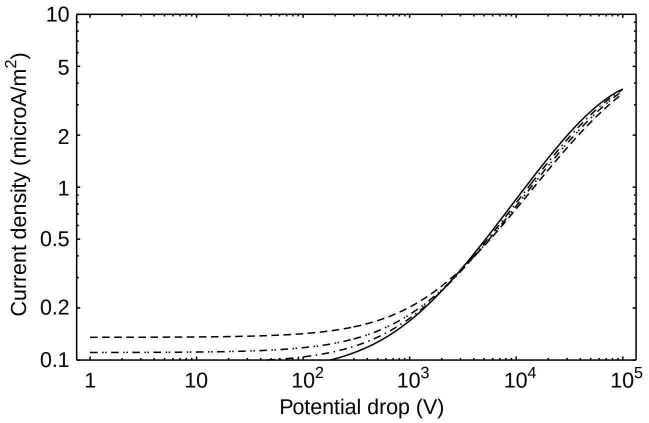

Fig. 3. Comparison of different κ values for the exact kappa current density formula; $\kappa = 2$ (*dot*), $\kappa = 3$ (*long-short-short dash*), $\kappa = 5.5$ (*dash-dot*), $\kappa = \infty$, i.e. Maxwellian (*solid*)

If we write the second approximation, Eq. (16), as $j = KV$, and compare with the corresponding Maxwellian result Eq. (11) putting $T = T_\parallel = T_\perp$ we can identify:

$$K^\kappa = \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\kappa^{3/2}} K^{\mathrm{BM}} \quad , \tag{17}$$

which differs from Eq. (14) of Pierrard (1996), who has $\kappa^{1/2}(\kappa-1)$ instead of $\kappa^{3/2}$ in the denominator. Our K^κ is always smaller than the corresponding K^{BM} (Fig. 5), whereas Pierrard's K^{Kappa} is larger than the Maxwellian K. For large κ values our results (and those of Pierrard's) approach the Maxwellian results, as they should.

Contrary to the Maxwellian case, the energy flux formula given by Eq. (7) applied to the kappa distribution yields to integrals which we cannot do analytically. However, we have already pointed out that in almost all practical cases it suffices to compute to first order in x. In the case of bi-Maxwellian distribution we first computed the exact current density and energy flux formulas, Eqs. (8) and (9), and then made the series expansion. It is,

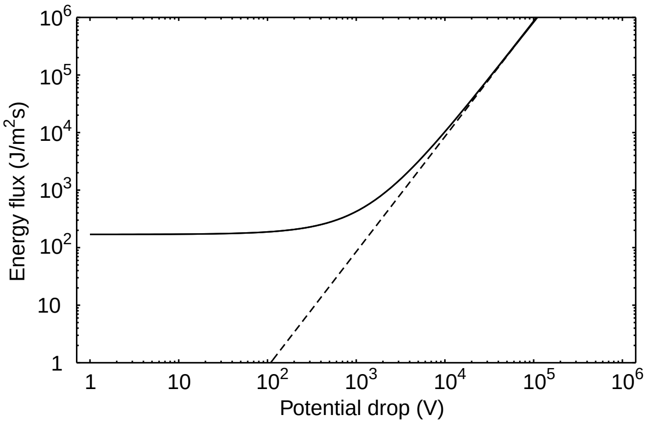

Fig. 2. Same as Fig. 1 but for the energy flux. In this case the first approximation (*dash*) is indistinguishable from the exact result (*solid*). The second approximation (*dot*) is still notably different for small energies

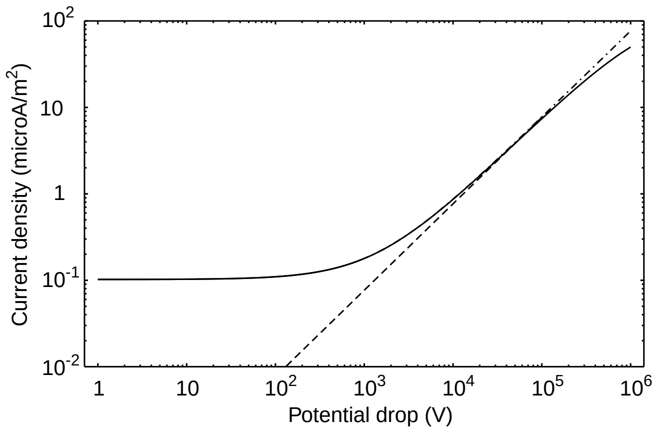

Fig. 4. Same as Fig. 1 but for the kappa distribution ($\kappa = 4$). The overlapping of the curves is similar to Fig. 1

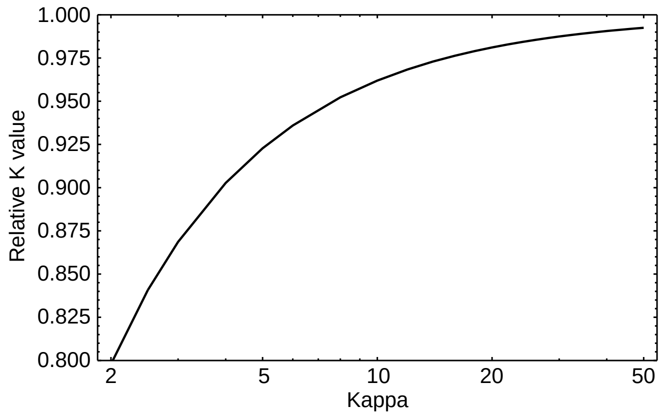

Fig. 5. Dependence of K^κ/K^{BM} on κ. Unity corresponds to the Maxwellian value. For large κ we recover the Maxwellian results as we should

however, also possible to utilize the approximation $x \ll 1$ first, as follows. In Eq. (7) the inner integration limit $W_\perp^{max}$ is proportional to x by Eq. (5). To first order in x, the value of the inner ($W_\perp$) integral is thus given by $W_\perp^{max}$ times the value of the integrand at $W_\perp = 0$:

$$\varepsilon = 2\pi\left(\frac{1}{2}\right)\left(\frac{B_i}{B_m}\right)\int_0^\infty W_\perp^{max} f(W_\parallel, 0)\sqrt{\frac{2}{m^3 W_\parallel}} \times v_\parallel(W_\parallel + eV)dW_\parallel + O(x)^2 \quad . \tag{18}$$

Physically, this approximation means that we approximate the distribution function inside the entire loss cone by the value of the distribution function at zero pitch angle. Since the loss cone is very narrow in the magnetospheric source region, this is a good approximation. We have rederived the bi-Maxwellian results, Eqs. (10) and (12), using this approximation to verify that it indeed yields the same results as the more direct method used in the preceding.

The energy flux formula of Eq. (18) applied to the kappa distribution gives the result (after utilizing the approximation $x \ll 1$ also elsewhere in the formula)

$$\varepsilon^\kappa_{\text{Approx.}} = \frac{N_e}{\sqrt{2\pi\kappa m T}\,\Gamma(\kappa - 1/2)} \times \left[2\kappa^2 T^2 \Gamma(\kappa-2) + 2e\kappa T V \Gamma(\kappa - 1) + e^2 V^2 \Gamma(\kappa)\right] \quad , \tag{19}$$

which is the kappa energy flux formula in the first approximation. This is a new result. Again, one can show that $\lim_{\kappa\to\infty} \varepsilon^\kappa_{\text{Approx.}} = \varepsilon^{BM}_{\text{Approx.}}$. Approximating this further by assuming a large potential drop relative to thermal energy, we obtain the second approximation formula

$$\varepsilon = \frac{e^2 N_e}{\sqrt{2\pi m T}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa - 1/2)\kappa^{3/2}} V^2 \quad . \tag{20}$$

Writing this in the form $\varepsilon = K^\kappa V^2$, we can see that the same K^κ can be identified both from the current density [Eq. (16) above] and the energy flux formula, Eq. (20).

In Fig. 6 we compare the first and second approximation energy fluxes in case of kappa distribution. The second approximation becomes invalid for small energies, as usual. The exact result is unfortunately not available, but since all other comparisons (Figs. 1–3) showed that the first approximation is almost indistinguishable from the exact result, there is every reason to believe that this is also the case for the kappa energy flux.

As a final note, the source plasma density N_e appearing in all the preceding formulas is not necessarily the true magnetospheric plasma density, because the electron loss cone filling during one bounce period is not necessarily complete. In other words, the starting point for our derivation was isotropic source plasma distribution function, which is the same as to assume complete loss cone filling by pitch angle scattering. It is very common, however, that the pitch angle scattering is incomplete for the electrons. This is seen, e.g., in the Freja study by Olsson *et al.* (1997), where the estimated effective source plasma densities were much lower than the true plasma density can possibly be.

3 Summary of results

We computed the current density and the energy flux for bi-Maxwellian and kappa distributions both exactly and in two approximations. The first approximation is almost generally valid and uses only $B_m \ll B_i$. In the second approximation we assume in addition that the thermal energy is much smaller than the acceleration potential V.

For the use with large-scale simulations at least, it is sufficient to consider magnetospheric electrons only as FAC carriers. Therefore we ignore ionospheric particles as well as magnetospheric protons, and we neglect gravity.

The results are valid for upward FAC regions. In downward FAC regions the classical theory predicts that the potential drop is approximately zero. Slight

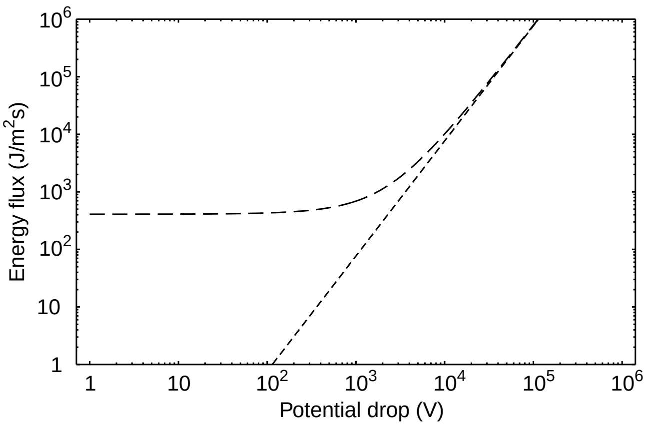

Fig. 6. Same as Fig. 4 but for the energy flux. The *dashed line* is the first approximation and the *dotted line* is the second approximation. The exact formula for the kappa energy flux is not known, so there is no solid line

modifications of this rule will in fact occur for very small potential drops which are possible to take into account by including the effect of ionospheric particles and magnetospheric protons (Lemaire and Scherer, 1983; Pierrard, 1996), but these are insignificant, at least as far as global ionosphere-magnetosphere coupling simulation work is concerned.

We summarize our new findings briefly.

1. In all example figures the first approximation curve was almost indistinguishable from the exact result. As the first approximation formulas are also practical to compute, they are useful for simulation work where an accurate current-voltage relationship is needed in both active and background regions. The second approximation is valid if the thermal energy is much smaller than the potential drop; it has thus been used for many observational studies of auroral activity (e.g., Lyons *et al*., 1979; Lu *et al*., 1991; Weimer *et al*., 1987; Sakanoi *et al*., 1995).

2. With the preceding assumptions, the current density formulas for kappa distribution were derived both exactly and in the two approximations.

3. We derived the kappa energy flux formulas in both first and second approximation.

4. We showed how the first approximation formulas can be derived even in cases where the exact result is not possible to compute analytically. This method must be used to derive the kappa energy flux formulas.

5. For the case of kappa distribution, the effective field-aligned conductance K is always smaller than for the bi-Maxwellian case. However, the difference is not very large and tends to unity when $\kappa \to \infty$.

6. For $\kappa \to \infty$ we recover the Maxwellian results for both current density and energy flux, as we should.

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